

Application of fractal techniques to the comparative evaluation of two methods of extracting channel networks from digital elevation models

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Abstract. The extraction of channel networks from digital elevation models was carried out using two different techniques: the “flow accumulation method” (which is based on the now almost traditional steepest descent flow routing) and the “multilevel skeletonization method” (which is based on curvatures in contours). Series of networks representing the same drainage basin were extracted with each of the methods using different extraction thresholds. Various stream ratios and fractal dimensions were determined from these networks based on three different hierarchical models. The study shows that for each extraction method, all ratios irrespective of hierarchical model exhibit similar variation with respect to extraction threshold. On the other hand, for the flow accumulation method of extraction, the different types of fractal dimensions vary in different ways with extraction threshold while in the case of the multilevel skeletonization method all the fractal dimensions decrease as extraction threshold increases. We attribute this difference in the behavior of fractal dimensions to the difference in approach between the two methods of extraction. Apparently the multilevel skeletonization method employs a more natural scheme.

1. Introduction

It has been recognized that drainage channel networks, like many other natural features, possess fractal qualities [Mandelbrot, 1977, 1983]. However, several attempts have thus far been made to establish the fractal dimension of channel networks, with conflicting results [La Barbera and Rosso, 1987, 1989; Tarboton et al., 1988; Helmlinger et al., 1993; Nikora and Sapozhnikov, 1993] (see also Phillips [1993] for further analysis). The origin of the problem is twofold. First, there is some disagreement regarding the relative relevance and the functional relationships of measures used for calculating fractal dimension (see Nikora and Sapozhnikov [1993] for a comparative evaluation). Second, the quality or suitability of channel networks upon which fractal computations are based is an issue of great importance. It is this second problem that we shall devote more attention to in this paper.

Channel networks usually are extracted either manually (from field data, images, or maps) or automatically (from digital data such as the digital elevation model (DEM)). Manual methods are generally time consuming and susceptible to subjectivity. Several researchers have pointed out a number of inadequacies associated with the manual extraction of channel networks [Mark, 1983, 1984; Morris and Heerdegen, 1988; Tarboton et al., 1991; Helmlinger et al., 1993]. In recent years, attention has been directed to automated network extraction techniques in view of their inherent advantages. Although they are not totally immune to errors, automated techniques are rapid, consistent, reproducible, and free from variable subjective influences. Moreover, as they operate in the digital mode, they generate data which are suitably adapted for numerical

analyses. None of these advantages applies to manual methods. However, one factor that is common to both methods is that the achievable resolution (that is, the size of the smallest features that can be extracted) depends on the scale of operation. In other words, the smaller the scale, the lesser the details. Thus in the case of automated extraction, features which are not represented in the DEM data owing to their small dimension relative to the sampling interval obviously cannot be extracted [Chorowicz et al., 1993]. Despite this implicit scale-imposed generalization, most algorithms explicitly include a trimming module which, after the main extraction process usually performed by the principle of steepest descent flow routing, tries to eliminate certain small details. The justification usually put forward is that allowance is being made for support (or contributing) areas for channel sources. Such exercises involve the application of various forms of thresholds [Morris and Heerdegen, 1988; Jenson and Dominique, 1988; Tarboton et al., 1991; Chorowicz et al., 1992, 1993] which when their values are changed, the density of the generated network changes.

This procedure, which hitherto has been the most common method used to extract channel networks automatically from DEMs, recently has been critically reexamined [Helmlinger et al., 1993; Montgomery and Fournelle-Georgiou, 1993; Costa-Cabral and Burges, 1994]. The reevaluation was approached from two perspectives. The first one concerns the effect of support area thresholds on the location of channel heads in extracted networks. Helmlinger et al. [1993] and Montgomery and Fournelle-Georgiou [1993] discussed different aspects of the application of thresholds, which depend on local topographic slope as opposed to the use of a constant threshold over an entire DEM, as is often the practice. Most of their results are demonstrated with examples. However, field evidence and further analysis regarding the relationship between local slope, channel head location, and support area are given by Montgomery and Dietrich [1994]. The second approach to the reeval-

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uation deals with the method of flow routing. *Montgomery and Foufoula-Georgiou* [1993] and *Costa-Cabral and Burges* [1994] highlighted the inadequacies in the common method of flow routing whereby flow simply is directed from a pixel to one of its eight nearest neighbors. One of the greatest weak points of this approach is its limitations in the extraction of divergent and convergent flows. Furthermore, *Costa-Cabral and Burges* [1994] discussed the various problems associated with some of the existing algorithms and proposed a new flow-routing algorithm with which some of the problems could be overcome.

More recently, a new algorithm employing the principle of multilevel skeletonization has been developed [*Meisels et al.*, 1995]. This algorithm does not perform flow routing explicitly. Rather, networks are implicitly derived by skeletonizing the DEM based on contours. Thresholds applied in this method represent degrees of contour curvatures rather than support areas.

In this paper we seek to make a comparative study of networks extracted by the methods of flow routing and multilevel skeletonization by means of fractal techniques. Networks are generated by both methods with different thresholds. Their fractal parameters are computed and subjected to detailed analysis.

2. Background

Even a visual assessment of channel networks shows that they exhibit statistical self-similarity, at least over limited areas and within the conventionally applicable range of scales. This implies that within a given area, a subnetwork when enlarged resembles its parent network. Therefore it follows that channel networks are fractal [*Mandelbrot*, 1977, 1983]. Generally, fractal properties of features are quantified using the so-called fractal dimension.

A number of works have been published on the characterization of the fractal properties and the determination of the fractal dimension D of channel networks. The most conventionally applied techniques include the ruler method, the box-counting method, and a computational method making use of Horton's ratios [*Tarboton et al.*, 1988; *Rosso et al.*, 1991]. Both the ruler and the box-counting methods are simple procedures often employed in many fields of study for determining the fractal dimensions of curves or features composed of curves [*Hastings and Sugihara*, 1993; *Roach and Fowler*, 1993].

These two methods involve similar steps which may be summarized as follows. In the ruler method the length of the feature of interest is measured with a ruler (or other length measuring scale such as dividers) set to various sizes, and the apparent length of the feature obtained with each measuring size is recorded. For instance, if a ruler of size r_i is used to measure the length of the feature in N_i steps, then its apparent length L_i equals $N_i r_i$. The fractal dimension of the feature is estimated from the slope of the plot of $\log(L_i)$ against $\log(r_i)$ as follows [*Feder*, 1988; *Roach and Fowler*, 1993]:

$$D_r = 1 - \text{slope} \quad (1)$$

where D_r denotes the fractal dimension by the ruler method. In the box-counting method, grids of different cell sizes (side lengths) are successively superimposed on the feature and the number of cells containing elements of the feature is counted. If we denote each grid size by r_i and the corresponding number of occupied cells by N_i , then the fractal dimension is the slope of the plot of $\log(N_i)$ against $\log(r_i)$ [*Feder*, 1988].

Nevertheless, when the ruler or the box-counting method is applied to channel networks the log-log plots in either case very often result in two distinct slopes instead of a unique slope [*Tarboton et al.*, 1988]. This tendency makes interpretation difficult. *Helminger et al.* [1993] drew attention to this problem. *Tarboton et al.* [1988] had earlier suggested that the smaller slope value would correspond to the fractal dimension of individual streams while the larger one would correspond to the fractal dimension of the network's branching structure. However, the hierarchical nature of channel networks which is considered an essential property in most modeling studies is ignored by the ruler and box-counting methods.

On the other hand, the method of Horton's ratios applies mainly to drainage systems. Although *Phillips* [1993] doubts the reliability of this method, and there is still some discussion in the literature as to the most appropriate measures to be involved in the computations, we think that it is more adaptive to channel networks (than the ruler or box-counting methods) since it recognizes their hierarchical structure. However, it is outside the scope of this work to discuss the suitability or otherwise of methods of determining fractal dimension. We shall use only the method of Horton's ratios in this work.

When the channels of a network are hierarchically ordered according to the system of either *Horton* [1945] or *Strahler* [1952], the following ratios may be established for channels of order ω (more conventionally referred to as streams of order ω):

$$R_B = N_{\omega-1}/N_{\omega} \quad (2)$$

$$R_L = L_{\omega+1}/L_{\omega} \quad (3)$$

$$R_A = A_{\omega+1}/A_{\omega} \quad (4)$$

R_B , R_L , and R_A represent bifurcation, length, and area ratios, respectively, where N_{ω} , L_{ω} , and A_{ω} respectively denote the number, average length, and average catchment area of streams of order ω . Equations (2) and (3) are based on the laws of stream numbers and stream lengths [*Horton*, 1945], and (4) on the law of stream areas [*Schumm*, 1956]. These laws are generally referred to as Horton's laws, and the ratios, which also are often referred to as Horton's ratios, are assumed to be constant within a drainage basin regardless of scale [*Rosso et al.*, 1991; *Helminger et al.*, 1993]. For a network, therefore, each can be estimated from the slope of the linear least squares fit to the corresponding Horton diagram [*Shreve*, 1966], which is the plot of the logarithms of N_{ω} , L_{ω} , or A_{ω} (as the case may be) against order ω . In the Horton's ratios method of fractal dimension computation, D is expressed as a function of two or more of the stream ratios R_B , R_L , and R_A . Nevertheless, there are variations in the specific formulas proposed by different authors [*Tarboton et al.*, 1988, 1990; *La Barbera and Rosso*, 1989, 1990; *Rosso et al.*, 1991] (see also *Nikora and Sapozhnikov* [1993] for a summary of these formulas). However, the relationship most frequently employed in the computation of fractal dimension is

$$D = \log R_B / \log R_L \quad (5)$$

Whereas *La Barbera and Rosso* [1987, 1989] derived this relation (5) for computing D as the fractal dimension of a network, *Tarboton et al.* [1990] held that the D computed from (5) represents just the fractal dimension of the network's branching structure and that it should be multiplied by the fractal dimension of individual channels in order to obtain the fractal dimension of the network. Although *La Barbera and Rosso*

[1990] later obtained a different formula, the right-hand side of (5) was still a factor. Also, in subsequent papers [Rosso *et al.*, 1991; Helmlinger *et al.*, 1993; Nikora and Sapozhnikov, 1993] (5) has been used either alone or as a factor in computing fractal dimension by the method of Horton's ratios and in most cases has served as a basis for conclusion. However, Nikora and Sapozhnikov [1993] contend that the existing Horton's ratios methods of fractal dimension computation are characterized by the assumption that channel networks are self-similar, which they observe is not always true. Based on the analysis of simulated river networks, they reported that when the total length of river networks is small, the networks exhibit self-similarity, while when they are large they exhibit self-affinity, and suggested that river systems should be associated more generally with self-affine rather than self-similar behaviors. They further recommended a larger set of parameters (than was hitherto used) for use in quantifying the fractal geometry of networks. Specifically, Nikora and Sapozhnikov [1993] and Nikora [1994] agree that (5) should be used for computing fractal dimension in a self-similarity situation and suggest that in the case of self-affinity, use should be made of two scaling indices: D_L , which is the same as D in (5), and D_W , which is given by

$$D_W = \log R_B / \log R_W \quad (6)$$

where R_W is the width ratio of the network expressed as

$$R_W = W_{\omega+1} / W_\omega \quad (7)$$

W_ω denotes the average width of subnetworks of order ω and can be determined from [Nikora and Sapozhnikov, 1993]

$$W_\omega = 2L_{\omega-1} \sin \varphi_{\omega-1} \quad (8)$$

in which $\varphi_{\omega-1}$ represents the angle between streams of adjacent Horton orders ω and $\omega - 1$. They recommend that D_L and D_W , referred to as scaling exponents for the longitudinal and transversal directions, be combined to compute Hurst index H (which characterizes the degree of self-affinity) and lacunarity dimension D_G (characterizing the degree of lacunarity or noncompactness of drainage basins) as follows:

$$H = D_L / D_W \quad (9)$$

and

$$D_G = (2D_L D_W) / (D_L + D_W) \quad (10)$$

Nikora and Sapozhnikov [1993] and Nikora [1994] further state that for self-similarity, $H = 1$ and $D_G = D_L = D_W = D$, while for self-affinity $H < 1$ and $D_L \neq D_W$. However, the parameters used for their computations are somewhat different from those usually derived on the basis of Horton's [1945] or Strahler's [1952] ordering schemes. Nikora and Sapozhnikov [1993] state that theirs represents a deterministic "fractal" model of the hierarchical structure of channel networks. They provide the following relationships between their parameters and those of Horton:

$$L_{f,\omega} \sim L_{H,\omega} \quad (11)$$

$$R_{f,L} = R_{H,L} \quad (12)$$

$$N_{f,\omega} = \frac{1}{L_{H,\omega}} \sum_{j=\omega}^{\Omega} L_{H,j} N_{H,j} \quad (13)$$

where ω denotes stream order, and the subscripts f and H indicate parameters belonging to the "fractal" and Horton's models, respectively. Thus $N_{H,\omega}$ and $L_{H,\omega}$ denote number and mean length, and $R_{H,L}$ length ratio of Horton streams of order ω , while $N_{f,\omega}$, $L_{f,\omega}$, and $R_{f,L}$ respectively denote the same parameters in the "fractal" model.

It is pertinent to examine the results so far published on the fractal dimension of channel networks. Tarboton *et al.* [1988] used the ruler, box-counting, and Horton's ratios methods to determine the fractal dimension of several river networks and concluded that there is a general agreement between the results, and that on the average their fractal dimension is 2. Rosso *et al.* [1991] used the Horton's ratios method to compute fractal dimensions from Hjelmfelt's [1988] data for eight rivers in Missouri (United States) and obtained values ranging from 1.444 to 1.649, with an average of 1.576 [Rosso *et al.*, 1991, p. 385]. Using the same method, they further computed the fractal dimension of 30 river networks around the world and reported a wide variation in the values (which range from around 1.4 to 2). They concluded that it is not right to assign an invariant fractal dimension to all river networks. Nikora and Sapozhnikov [1993] computed H and D_G for several natural rivers and obtained average values of approximately 0.73 and 1.87, respectively. However, there seems to be a universal consensus of opinion that in all cases, the value of channel network fractal dimension (i.e., D or D_G) should not exceed 2. This is based on the consideration that in its usual planimetric (two-dimensional) representation on a map, the channel network can at most fill up a plane with dimension 2.

An important issue to consider is the nature of channel networks used in fractal studies and upon which research results are based. Real as well as simulated networks are used. Here we are more concerned with real networks because they represent the true nature of things. Real networks are either extracted manually (mainly from topographic maps) or automatically (mainly from DEMs). This paper deals with DEM-derived channel networks. The extraction of channel networks from the DEM is based on purely morphological considerations, but very often their final composition is dependent on arbitrarily chosen thresholds used to make provision for support areas of channel sources. Despite this potential uncertainty in network composition, DEM-derived channel networks are gaining much popularity as input data for analyzing and modeling hydrophysical and hydrological processes. It is therefore imperative that sufficient effort should be devoted to research in assessing such networks in order to try to identify reliable extraction techniques. This can be done by either comparing them with known standards (if they exist) or by any other means. In this paper we seek to comparatively evaluate DEM-derived networks through fractal analysis. Tarboton *et al.* [1988] determined fractal dimensions for channel networks extracted with different support area thresholds and observed that the more detailed the network (i.e., the smaller the threshold), the smaller the scale above which it is space filling (with fractal dimension $D = 2$). Helmlinger *et al.* [1993] performed a similar work and concluded that not only is it difficult to determine the optimal threshold support area to apply, but also that applying a constant support area is inappropriate since, according to their findings, support area should depend on local slope.

A very recent work by Meisels *et al.* [1995] on multilevel skeletonization of DEMs (described in the next section) features the extraction of channel networks in which channel

source locations are determined not by the application of artificial threshold support areas but intrinsically on the basis of the degree of curvature of contour cusps which is related to local slope. In this work we study the fractal properties of channel networks extracted by this method with different degrees of curvature as well as those extracted by the flow accumulation method with different support area thresholds for comparison. Different models, covering the use of a wide range of parameters, will be used.

3. Extracting Channel Networks From DEM Data

Several algorithms have been proposed in the literature for the extraction of channel (or valley) networks from DEM data [e.g., Collins, 1975; Mark, 1984; O'Callaghan and Mark, 1984; Band, 1986; Douglas, 1986; Jenson and Domingue, 1988; Morris and Heerdegen, 1988; Riazanoff et al., 1988; Seemuller, 1989; Fairfield and Leymarie, 1991; Freeman, 1991; Quinn et al., 1991; Tarboton et al., 1991; Chorowicz et al., 1992; Costa-Cabral and Burges 1994]. A categorization of some of these in accordance with the principles they use has been made by Fairfield and Leymarie [1991] and Costa-Cabral and Burges [1994]. They designated "D8" the category of algorithms in which flow is routed from a pixel simply to the one of its eight nearest neighbors, which constitutes the steepest descent. Most of the existing algorithms belong to this group with slight differences. Another category, termed "Rho8," is represented by the algorithm proposed by Fairfield and Leymarie [1991], which is a modification of D8 in which the direction of flow routing is stochastically determined based on the slope aspect. The work of Lea [1992] represents a third category, in which flow is routed on the basis of aspect angle but not stochastically. Then there is the multiple-flow category in which the flow from a pixel is partitioned among its lower elevation neighbors according to a certain proportion [Freeman, 1991; Quinn et al., 1991]. The algorithm proposed by Costa-Cabral and Burges [1994] forms a different category in that it enables the modeling of parallel, divergent, and convergent flows depending on the topography. All the above algorithms explicitly perform flow routing to accumulate flow before the final network is extracted based on a flow accumulation threshold.

On the other hand, a newly developed multilevel skeletonization technique [Meisels et al., 1995] is based on contours; it neither routes nor accumulates flow, and channel head location is implicitly determined based on contour curvature threshold. The extraction of channel networks on the basis of contour curvatures has long been advocated even as far back as the era of manual extraction [Morisawa, 1957]. This idea was implemented digitally by Moore et al. [1988] and Moore and Grayson [1991] using digitized contours. Meisels et al. [1995] use a DEM to achieve this principle of contour-curvature-guided channel network extraction.

In this work we comparatively evaluate networks extracted by this newly developed multilevel skeletonization technique and the flow accumulation technique. In the case of the latter technique the D8 algorithm will be used, as it is the most widespread. We give a brief description of the two techniques in the following subsections. We use the conventional type of DEM, which is a matrix of numbers representing elevations of points at regular square grid intervals. Our DEM represents subwatershed 11 of the USDA-ARS (United States Department of Agriculture-Agricultural Research Service) Walnut

Gulch Experimental Watershed, Arizona. The grid interval is 30 m.

3.1. Flow Accumulation Method

This method is described in detail by O'Callaghan and Mark [1984] and Jenson and Domingue [1988]. It involves a number of steps. In the first step the DEM is either smoothed or the pits (or depressions) in it are filled to facilitate flow routing. Depressions are defined as areas completely surrounded by pixels of higher elevation. A single-pixel depression is filled by simply replacing its value by that of its lowest neighbor. However, depressions may comprise more than one pixel. Rendering the DEM completely depressionless involves a more intricate procedure (see Jenson and Domingue [1988] for detailed description). The second step involves the determination of flow directions for every pixel of the DEM. This is based on the analysis of the slope relationship between each pixel and its eight nearest neighbors. Flow direction is represented by a code indicating the relative location of the neighbor into which water would flow from the central pixel. Usually this is the neighbor which presents the steepest descent. In the third step, flow accumulation is derived from the flow direction data set. The flow accumulation for each pixel is represented by the number of pixels flowing into it, directly or indirectly. The final step of the extraction process is the realization of the channel network from the flow accumulation data set. This is achieved by simply selecting only pixels whose flow accumulation values are greater than or equal to a chosen threshold to constitute the network. This threshold is sometimes referred to as the threshold support area [e.g., Tarboton et al., 1988, 1991] since number of pixels is in this case related to area. Figures 1a-1f show, respectively, networks extracted by the flow accumulation method with support area thresholds of 5, 20, 50, 100, 150, and 200. Networks extracted with small threshold values in this method contain areas constituted by clusters of pixels without proper definition of channels, as can be observed in the one extracted with the threshold value of 5 (Figure 1a). This network is not suitably disposed for fractal analysis and consequently will not be used for subsequent computations to be carried out in our work because it will present difficulties to the automatic identification of flow lines. It is shown here in order to illustrate the problem of channel definition at low thresholds using this method.

3.2. Multilevel Skeletonization Method (A Contour-Based Technique)

This method, recently developed by Meisels et al. [1995], is based on the principle of parametrizable skeletonization of multilevel images proposed by Riazanoff et al. [1990]. In order to extract channel networks from a DEM, the algorithm begins from the highest areas and erodes the DEM downhill until the main skeleton of the channel network is obtained. This is based on the assumption that channels should situate at the topographically lowest parts of a valley bottom. The erosion of a pixel is achieved by replacing its actual value with a constant background value (usually 0, assuming that this is not a valid elevation value in the original DEM). In this type of work, nonbackground pixels may be referred to as "object" pixels. An object pixel with at least one (and at most three) background pixel(s) among the four nearest neighbors (related to it in the vertical or horizontal sense) may be referred to as "contour" pixel. Pixels linking two object parts are referred to as "connection" pixels. In this skeletonization process, only con-

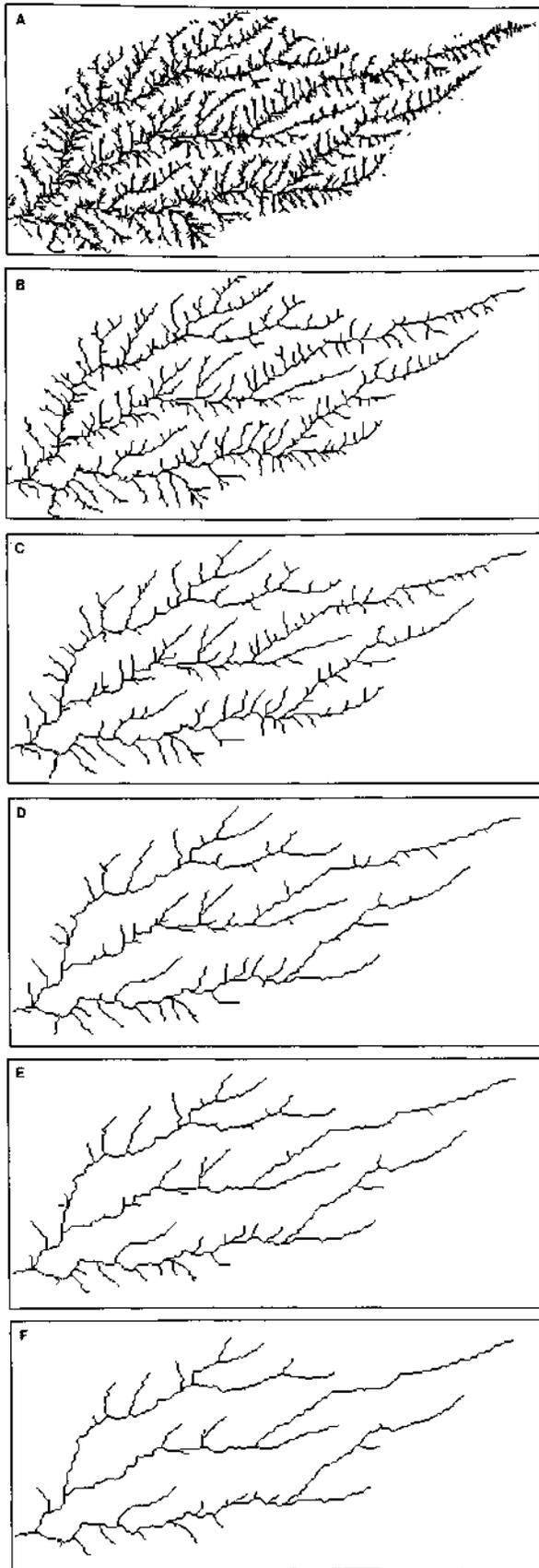


Figure 1. Channel network extracted from a DEM representing the subwatershed 11 of the USDA-ARS Walnut Gulch Experimental Watershed (Arizona, United States) using the flow accumulation method with support area thresholds of (a) 5, (b) 20, (c) 50, (d) 100, (e) 150, and (f) 200.

four pixels may be eroded based on the local contour curvatures. Contour curvature at a pixel, represented by a parameter called the "convexity," K , is roughly estimated by the highest number of contiguous background neighbors among its eight nearest neighbors. Thus values of K are integers and exist only in the range of 1 to 8. Usually, the user supplies a curvature (or convexity) threshold K_T such that pixels at which curvature is equal to or greater than this threshold cannot be eroded. Indeed, results are meaningful only for five K_T values (from 3 to 7).

In the skeletonization algorithm, first all local maxima (pixels higher than all their eight neighbors) throughout the DEM are assigned the background pixel value. Subsequently, the image is processed according to elevation values from the maximum to the minimum elevation. For each elevation value all object pixels with the current elevation are examined, and a pixel is eroded if it satisfies all the following conditions: (1) it is a contour pixel; (2) its convexity K is less than the threshold K_T ; (3) it was not a connection point before the preceding iteration; and (4) it is not currently a connection point. The whole image is processed repeatedly with the current elevation until no more pixels can be eliminated in the entire image. Then the next lower elevation value is processed.

After processing all the elevation values the entire process is performed once again in order to connect unconnected segments. However, to ensure proper channel connectivity, depressions are filled in the original DEM before this second stage of skeletonization is embarked upon. Depression filling is carried out according to elevation values from minimum to maximum, with an iteration at each elevation value. All pixels belonging to a depression are assigned a special constant value, called a "lake." This value should be lower than the overall minimum elevation (usually the minimum elevation minus 1) such that they will correspond to the last elevation value to be processed during skeletonization. The second stage of skeletonization is entirely the same as the first except that in considering a pixel for elimination, the convexity condition (condition 2) above is not applied. At the end of this second stage the resulting flow channels are well connected, although there may be a few flow loops caused by the erroneous connection of certain flow paths in the upstream sense. However, the algorithm incorporates certain functions which go further to eliminate these loops based on techniques referred to as flow path enumeration and traversal. Figures 2a-2d show, respectively, channel networks extracted by the multilevel skeletonization method with curvature threshold values of 3, 4, 5, and 6.

4. Determination of Network Fractal Parameters

All extracted networks are represented as binary raster images in which channel pixels have value 1 while background pixels have value 0. We have developed an algorithm to determine the network fractal parameters automatically. The procedure involves network tracing, ordering, parameter extraction, and fractal computation. Before the image containing networks is submitted to the algorithm, the value of the terminal pixel representing the outlet of each network to be processed is changed from 1 to a large value such as 100. This is to enable the algorithm to recognize network outlets because network tracing begins at the outlet and moves toward the sources, passing through the branches [Ichoku and Chorowicz, 1994].

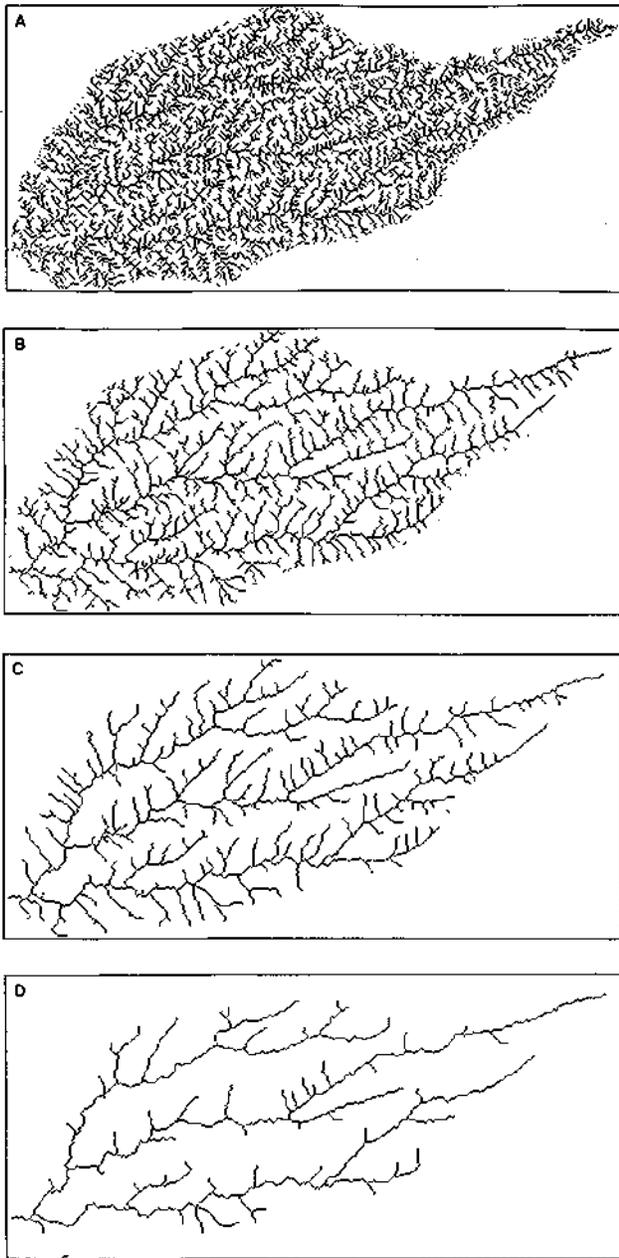


Figure 2. Channel network extracted from a DEM representing the subwatershed 11 of the USDA-ARS Walnut Gulch Experimental Watershed (Arizona, United States) using the multilevel skeletonization method with convexity thresholds of (a) 3, (b) 4, (c) 5, and (d) 6.

Starting from the outlet pixel, each link is traced by moving recursively from pixel to pixel. When a junction is reached, the current link ends and each branch is traced in a similar manner. This process continues until the entire network is traced. Every traced link is assigned an identification number and its details are entered in an array against this number. The details include the row/column coordinates of its beginning and end, its length, its direction, and the identification numbers of its branches and its downstream link, as well as codes indicating how it is connected to them. Details of the procedures for determining stream lengths and directions are given by *Ichoku*

and *Chorowicz* [1994]. After tracing, the network is ordered according to the ordering scheme of *Strahler* [1952].

Subsequently, the following network parameters are determined: number of streams N_ω and mean length of streams L_ω of each order ω . Also, junction angles φ are computed from stream directions and classified according to the orders of streams subtending them. Since ordering by the Horton scheme was not performed, due to the fact that it involves subjective decisions [*Shreve*, 1966, p. 18] which are difficult to implement in an algorithm, the foregoing parameters in the Horton scheme were derived from their equivalents in the *Strahler* scheme. Horton stream numbers N_H can be derived from *Strahler* stream numbers N_S as follows [*Shreve*, 1966, p. 23]:

$$N_{H,\omega} = N_{S,\omega} - N_{S,\omega+1} \quad (14)$$

where $N_{H,\omega}$ and $N_{S,\omega}$ represent number of streams of order ω with reference to the Horton and *Strahler* schemes, respectively. We estimated average lengths of Horton streams from the following relationship:

$$L_{H,\omega} = \sum_{i=1}^{\omega} L_{S,i} \quad (15)$$

where $L_{H,\omega}$ and $L_{S,\omega}$ represent average lengths of streams of order ω in the Horton and *Strahler* schemes, respectively. The stream numbers $N_{f,\omega}$ and average lengths $L_{f,\omega}$ of the "fractal" model proposed by *Nikora and Sapozhnikov* [1993] are further derived from (13) and (11), respectively. The average angle φ_ω subtended by Horton streams of adjacent orders ω and $\omega + 1$ was approximated by the mean of angles subtended by *Strahler* streams of orders ω and ω and of orders ω and $\omega + 1$. This is based on the knowledge that most junctions between *Strahler* streams of orders ω and ω correspond to a junction between Horton streams of orders ω and $\omega + 1$. Although the remaining few correspond to junctions between ω and higher orders, we found empirically that the effect of their inclusion in the computation of the average angular value is negligible. Average widths $W_{f,\omega}$ are computed from (8) using $L_{f,\omega}$ and φ_ω for $2 \leq \omega \leq \Omega$, where Ω denotes network order or the maximum order in a network.

Stream ratios R_B , R_L , and R_W are computed from least squares equations relating the logarithms of N_ω , L_ω , and W_ω , respectively, to ω . We computed bifurcation $R_{S,B}$ and length $R_{S,L}$ ratios for *Strahler* streams; bifurcation $R_{H,B}$ and length $R_{H,L}$ ratios for Horton streams; and bifurcation $R_{f,B}$, length $R_{f,L}$, and width $R_{f,W}$ ratios for the "fractal" model of *Nikora and Sapozhnikov* [1993]. Note that $R_{f,L}$ is equal to $R_{H,L}$, according to (12). Tables 1 and 2 show the values of the ratios computed from networks derived by the flow accumulation and multilevel skeletonization methods, respectively.

Finally, the various ratios are used for computing different fractal exponents. The fractal dimension D (equation (5)) has been computed from pairs of bifurcation and length ratios belonging to three different hierarchical models. It is denoted by D_S if computed from *Strahler* stream ratios $R_{S,B}$ and $R_{S,L}$; by D_H if computed from Horton stream ratios $R_{H,B}$ and $R_{H,L}$; and by D_L if computed from the "fractal" model stream ratios $R_{f,B}$, $R_{f,L}$ [*Nikora and Sapozhnikov* 1993]. D_W , H , and D_G are computed from (6), (9), and (10), respectively, using the "fractal" model stream ratios $R_{f,B}$, $R_{f,L}$, and $R_{f,W}$. The computed values of the fractal exponents are shown in Tables 3 and 4 for networks derived by the flow accumulation and multilevel skeletonization methods, respectively.

Table 1. Stream Ratios (Based on Strahler, Horton, and "Fractal" Hierarchical Models) for Networks Extracted by the Flow Accumulation Method

Support Area Threshold, in Pixels	Network Order, Ω	Horton's Model*		Strahler's Model†		"Fractal" Model‡		
		$R_{H,B}$	$R_{H,L}$	$R_{S,B}$	$R_{S,L}$	$R_{f,B}$	$R_{f,L}$	$R_{f,W}$
20	5	3.826	3.241	3.880	2.798	4.728	3.241	3.375
50	4	5.102	5.468	5.231	5.167	6.975	5.468	5.876
100	4	3.793	3.759	3.958	3.022	5.117	3.759	4.109
150	4	3.192	3.332	3.410	2.544	4.421	3.332	3.681
200	4	2.966	3.045	3.070	2.219	3.852	3.045	2.941

*Horton [1945].
 †Strahler [1952].
 ‡Nikora and Sapozhnikov [1993].

5. Results and Discussion

In this section we shall examine the characteristics of the networks extracted with different thresholds by the two methods described above as well as the scaling parameters (ratios and fractal exponents) obtained from them. A visual comparison of the extracted networks (Figures 1 and 2) shows that the multilevel skeletonization method appears to have an advantage over the flow accumulation method in terms of channel definition, especially at smaller extraction thresholds. For instance, even though Figure 2a is highly detailed, all channels are very clearly defined, whereas by comparison Figure 1a, which is not as detailed, portrays very poor channel definition. *Montgomery and Foufoula-Georgiou* [1993] show that the use of low thresholds in the flow accumulation method results in the problem of "feathering," whereby channels extend into planar or divergent hillslopes. This is probably due to the fact that in the flow accumulation method (especially the D8 type), flow is routed from every part of the DEM simply according to the steepest descent; thus all points meeting the specified support area threshold are extracted irrespective of whether the resulting flow lines correspond to places of landscape dissection as represented by valleys and channels (see *Montgomery and Dietrich* [1994] for a discussion on the differences between valleys and channels). In contrast, the multilevel skeletonization method locates flow lines only up to places where the presence of landscape dissection is identified by curvatures in contours, the degree of curvature meeting the specified threshold. A more global comparison based on the scaling parameters of the networks extracted by the two methods will be pursued below.

Figure 3 shows plots of the different ratios computed in this work against extraction thresholds for networks extracted by flow accumulation (Figure 3a) and multilevel skeletonization

(Figure 3b) methods. In either case, graphs of all ratios have almost the same shape (with peaks occurring at the same threshold values in each case). In order to verify whether this similarity in variation observed in our case was just a coincidence, we tried plotting similar graphs for the stream ratios determined from networks extracted from DEMs representing other basins; we observed that in general, using either of the two methods of extraction described in this work, for a given basin, graphs of the different ratios (against extraction thresholds) have almost the same shape, although they do not necessarily have single prominent peaks like ours. This is also apparent, for instance, in the plot of Strahler bifurcation and length ratios (Figure 4) obtained from the data of South Fork Smith River, California (Figure 4a); Schoharie Creek, New York (Figure 4b); and Big Creek, Idaho (Figure 4c), basins presented in Table 3 of *Helmlinger et al.* [1993]. Therefore it appears that for each of the two extraction methods used in this work, for a given drainage basin, as the extraction threshold changes, all stream ratios regardless of hierarchical model vary in an almost identical manner.

The computed scaling exponents D_S , D_H , D_L , D_W , H , and D_G have also been plotted against extraction thresholds for the two methods of extraction. These are shown in Figures 5a and 5b for networks extracted by the flow accumulation and multilevel skeletonization methods, respectively. It can be observed that the dimensions computed from the flow accumulation networks behave in different ways. On the other hand, in the case of networks extracted by the multilevel skeletonization method, all dimensions (excluding H , which is not a dimension and for which we obtained values always close to 1) have almost the same behavior: all tending to descend continuously with increasing extraction threshold. It is very obvious that in

Table 2. Stream Ratios (Based on Strahler, Horton, and "Fractal" Hierarchical Models) for Networks Extracted by the Multilevel Skeletonization Method

Convexity Threshold	Network Order, Ω	Horton's Model*		Strahler's Model†		"Fractal" Model‡		
		$R_{H,B}$	$R_{H,L}$	$R_{S,B}$	$R_{S,L}$	$R_{f,B}$	$R_{f,L}$	$R_{f,W}$
3	6	4.425	3.039	4.381	2.816	5.034	3.039	3.221
4	5	4.634	3.485	4.752	3.228	5.744	3.485	3.550
5	4	5.490	4.134	5.721	3.717	7.065	4.134	4.319
6	4	3.497	3.913	3.693	3.317	5.001	3.913	3.806

*Horton [1945].
 †Strahler [1952].
 ‡Nikora and Sapozhnikov [1993].

Table 3. Fractal Exponents (Based on Strahler, Horton, and "Fractal" Hierarchical Models) for Networks Extracted by the Flow Accumulation Method

Support Area Threshold, in Pixels	Network Order, Ω	Horton's Model,* D_H	Strahler's Model,† D_S	"Fractal" Model‡			
				D_L	D_W	H	D_G
20	5	1.141	1.318	1.321	1.277	1.035	1.299
50	4	0.959	1.007	1.143	1.097	1.042	1.120
100	4	1.007	1.244	1.233	1.155	1.067	1.193
150	4	0.964	1.314	1.235	1.141	1.083	1.186
200	4	0.977	1.408	1.211	1.250	0.969	1.230

*Horton [1945].

†Strahler [1952].

‡Nikora and Sapozhnikov [1993].

both methods, the higher the extraction threshold, the lesser the detail expressed by the extracted network, and vice versa. Voss [1988] demonstrates that the fractal dimension of a fractal object increases with increasing detail. This was clearly expressed in his statement, "The fractal dimension determines the relative amounts of detail or irregularities at different distance scales. Surfaces with a larger D seem rougher." [Voss, 1988, p. 31]. He illustrated this direct relationship between fractal dimension D and detail both for curves, with the aid of samples of fractional Brownian motion traces in his Figure 1.12, and for surfaces in his Plates 8–13. Several authors have simulated curves and surfaces of varying fractal dimension, and it is observed that the larger the fractal dimension, the more complex the simulated features [Goodchild, 1980, 1988; Saube, 1988; Lam, 1990]. In fact, it has been noted that a large "set of applications utilizes the fractal dimension as an index for describing the complexity of curves and surfaces." [Lam, 1990, p. 188]. Therefore in the case of channel networks extracted in this work, since detail decreases with increasing threshold, fractal dimension should also be expected to decrease with increasing threshold. This has been the case with networks extracted by the multilevel skeletonization but not for those extracted by the flow accumulation method. It is remarkable that for networks issuing from the former extraction method, even though the various types of fractal dimensions (D_S , D_H , and D_L) have been determined on the basis of different hierarchical models, they have all behaved similarly. The multilevel skeletonization method of network extraction therefore appears to give a more natural result.

The difference between the behaviors of the computed fractal dimensions of the two groups of networks with respect to extraction threshold can be explained by the difference in the nature of these thresholds. The effect of thresholds in network

extraction is to determine the location of channel heads with respect to the drainage divide. Field evidence shows that the location of a channel head is a function of the local valley slope [Montgomery and Dietrich, 1992]. The support area threshold used in the flow accumulation method applies equal area condition to all channel heads whereas the convexity threshold used in the multilevel skeletonization method is based on contour curvature, which is a function of local terrain slope. Therefore the multilevel skeletonization method adapts more naturally to topographic variabilities and networks generated with it are likely to bear a closer resemblance to the true channel network.

6. Conclusion

In this work we have applied fractal techniques to investigate the effects of thresholds on channel networks extracted by two different automated techniques: the flow accumulation and the multilevel skeletonization methods. The assessment has been performed on the basis of scaling parameters computed from these networks, namely, various types of Horton's ratios and fractal exponents based on different hierarchical models. With reference to the ratios, our study shows that using either method, for a given DEM representing a given drainage basin, all the ratios vary with network extraction threshold in an almost identical manner. As regards the fractal exponents, we have observed that in the networks extracted by the flow accumulation method, the different fractal dimensions behave differently with respect to extraction threshold, while in those extracted by the multilevel skeletonization method they all behave similarly, decreasing as extraction threshold increases (and detail decreases). The latter case is believed to be the appropriate pattern because evidence has shown that at con-

Table 4. Fractal Exponents (Based on Strahler, Horton, and "Fractal" Hierarchical Models) for Networks Extracted by the Multilevel Skeletonization Method

Convexity Threshold	Network Order, Ω	Horton's Model,* D_H	Strahler's Model,† D_S	"Fractal" Model‡			
				D_L	D_W	H	D_G
3	6	1.338	1.427	1.454	1.382	1.052	1.417
4	5	1.228	1.330	1.400	1.380	1.015	1.390
5	4	1.200	1.328	1.378	1.336	1.031	1.357
6	4	0.918	1.090	1.180	1.204	0.980	1.192

*Horton [1945].

†Strahler [1952].

‡Nikora and Sapozhnikov [1993].

stant scale, fractal dimension has a direct relationship with detail. This situation has led us to conclude that the multilevel skeletonization method gives a more natural performance because its threshold adapts to local terrain slope in locating channel heads during extraction, whereas the threshold used in the flow accumulation method does not. Moreover, in the flow accumulation method applicable thresholds range from 1 to the maximum flow accumulation value in a given DEM (which is of the order of 30,000 for our DEM), thereby making choice difficult and the significance of the outcome uncertain. But in the case of the multilevel skeletonization method the range of applicable thresholds is very narrow (only from 3 to 7), and thus not only is choice facilitated but generated results are also meaningful (since the extraction is based on contour curvatures, in conformity with the conventional principle of extracting networks from topographic maps).

The result obtained in this work further supports the observation that channel source support area, and consequently the location of channel heads, depends on local slope [Montgomery and Dietrich, 1988, 1992; Helmlinger et al., 1993]. In fact, for one of the drainage basins (Schoharie Creek, New York) they studied, Helmlinger et al. [1993] observed that with an equal

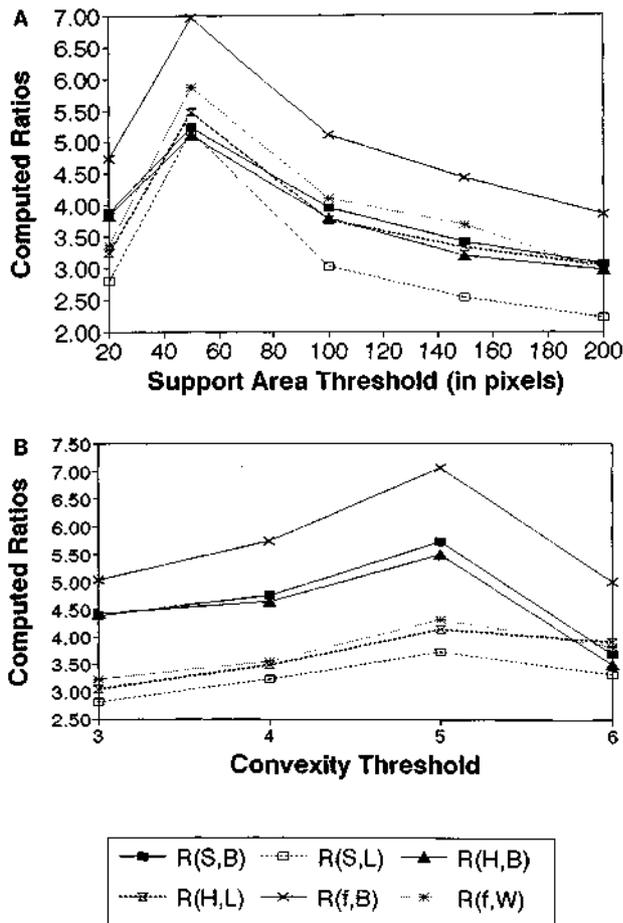


Figure 3. Plots of different kinds of stream ratios (based on Strahler, Horton, or "fractal" hierarchical models) against extraction threshold for networks extracted by (a) the flow accumulation and (b) the multilevel skeletonization methods. $R(S,B) = R_{S,B}$; $R(S,L) = R_{S,L}$; $R(H,B) = R_{H,B}$; $R(H,L) = R_{H,L}$; $R(f,B) = R_{f,B}$; and $R(f,W) = R_{f,W}$.

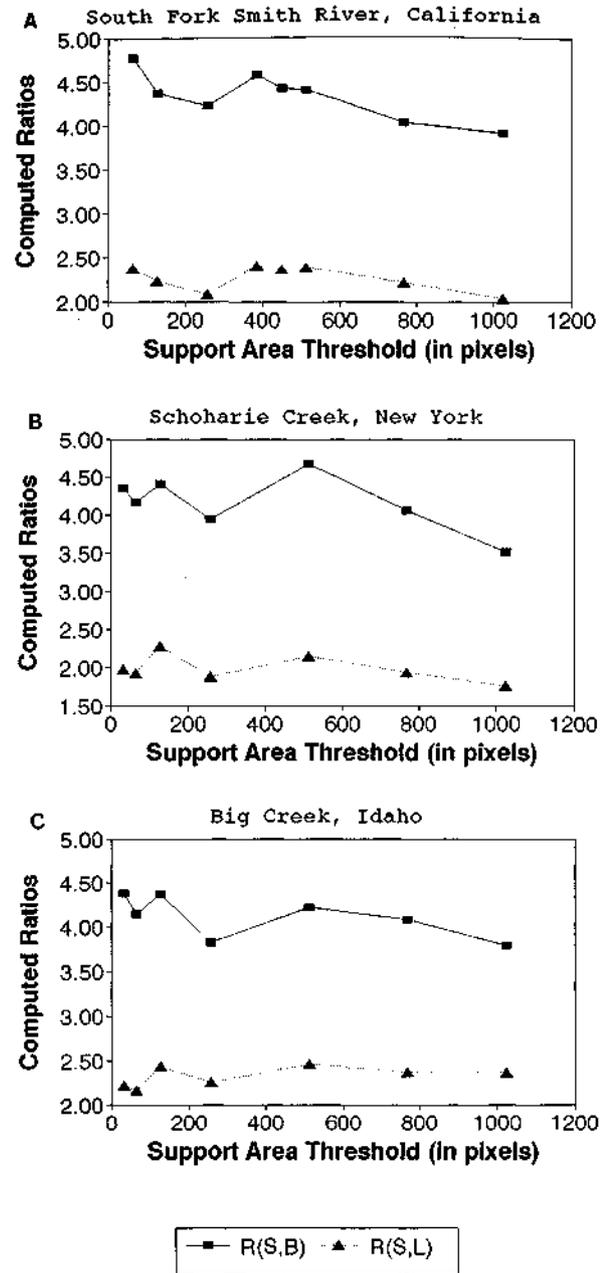


Figure 4. Plots of bifurcation and length ratios (based on the Strahler hierarchical model) against extraction threshold for networks extracted by the flow accumulation method for DEMs representing (a) South Fork Smith River, California; (b) Schoharie Creek, New York; and (c) Big Creek, Idaho, basins (data presented in Table 3 of Helmlinger et al. [1993]). $R(S,B) = R_{S,B}$; $R(S,L) = R_{S,L}$.

support area threshold, the computed fractal dimension values were unrealistic, but when they applied a slope-dependent area threshold to the same basin, the computed fractal values improved in line with their expectations. It is therefore imperative that algorithms used in the extraction of channel networks from DEMs should be equipped with the capability for adapting the location of channel heads to local terrain slope.

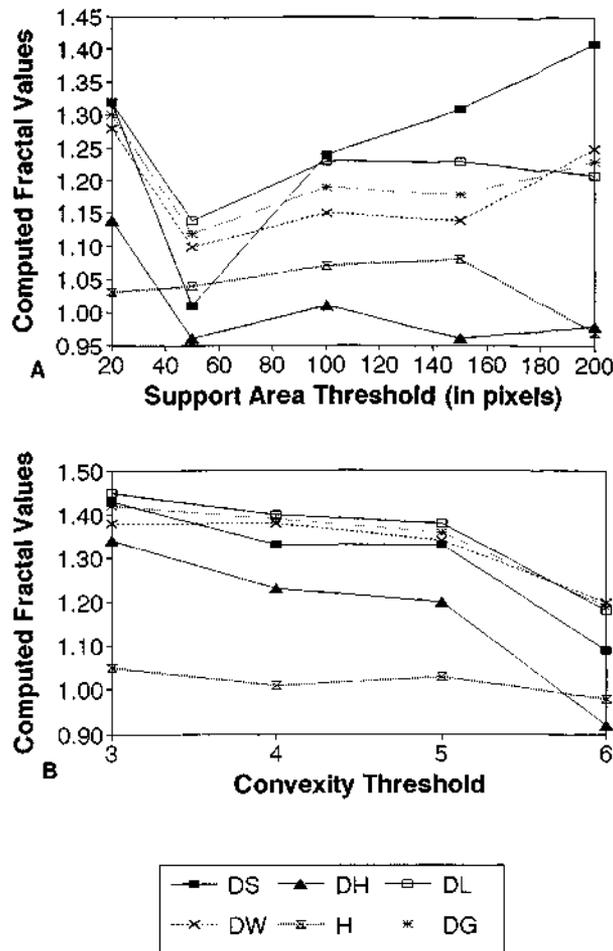


Figure 5. Plots of different kinds of fractal exponents (based on Strahler, Horton, or "fractal" hierarchical models) against extraction threshold for networks extracted by (a) the flow accumulation and (b) the multilevel skeletonization methods. DS = D_S ; DH = D_H ; DL = D_L ; DW = D_W ; H = H ; and DG = D_G .

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